A NOTE ON SOME BALANCED GENERALIZED TWO-WAY ELIMINATION OF HETEROGENEITY DESIGNS

By
DAMARAJU RAGHAVARAO
Punjab Agricultural University, Ludhiana.
(Received in December, 1969)

1. INTRODUCTION :

We follow the notation of Aggarwal (196: *a, b, c*) for the analysis and constructions of generalized two-way elimination of heterogeneity designs. In the references at the end of this paper many balanced and partially balanced designs were given. In this note we shall give some balanced designs not included in the literature so far and to this end we need the concept of generalized systems of distinct representatives (SDR).

Let $s_1, s_2, \ldots s_n$ be n subsets not necessarily disjoint of a given finite set S. Then $(0_1, 0_2, \ldots\ldots, 0_n)$ will be called a $(m_1, m_2, \ldots\ldots, m_n)$ SDR if

(*i*) $o_i \subset S_i,\ i=1, 2, \ldots, n$

(*ii*) Card $(o_i)=m_i,\ i=1, 2, \ldots, n$, and

(*iii*) $o_i \cap o_j=\phi,\ i\neq j=1, 2, \ldots, n$

Then Aggarwal (1966 *d*) has proved that a necessary and sufficient condition for the existence of a $(m_1, m_2, \ldots, m_n)$ SDR for the sets $S_1, S_2, \ldots, S_n$ is that, wherever $1\leqslant K\leqslant n$ and $i_1<i_2<\ldots<i_k\leqslant n$, then

$$\text{Card}\ \ (T_{i_1} \cup T_{i_2} \cup \ldots \cup T_{i_k}) \geqslant \sum_{i=1}^{k} m_{i_j}$$

2. MAIN RESULTS

(A) Designs with

$$u=u'=v \quad \text{and} \quad L=M=N=J_v-I_v$$

The series of designs with $u=u'=v$ and $L=M=N=J_v-I_v$, where I_v is the identity matrix of order v and J_v is a vxv matrix with $+1$ everywhere, can be easily verified to be balanced for the row, column and treatment effects comparisons and this series belongs to class 2 designs of the three factor additive designs of Pothoff (1962). A method of constructing this series for v odd was given by Aggarwal (1966 *b*) and he gave trial and error solutions for the two even values 4 and 16 for v. We first establish that this series always exists for every v and give the constructed designs for $v=6$, 8 and 10.

Let us consider the v sets $R_1^*, R_2^*, \ldots, R_v^*$, where

$$R_i^*=\{1, 2, \ldots, i-1, i+1, \ldots, v\}.$$

Let $$R_i^{*\prime}=R_i^*-\{1\}.$$

Then we can easily verify that any k of the $R_i^{*\prime}$ contain between themself at least $k-1$ or k distinct symbols according as the set contains 0 or 1 symbols in the $(0, 1, \ldots, 1)-$SDR. Hence a $\{0,1,1,\ldots,1)$ SDR exists for the v sets $R_i^{*\prime}$. We arrange them in the first column such that the 1st position is blank and the rest of the positions are filled by 2, 3,..., v each of the treatments occurring exactly once. If γ_i occurs in the above SDR from $R_i^{*\prime}$, then we consider

$$R_i^{**}=R_i^*-\{\gamma_i\} \text{ for } i=1, 2\ldots, v.$$

Let $$R_i^{**\prime}=R_i^{**}-\{2\}.$$

Then once again we can easily show that any K of the $R_i^{**\prime}$ contain between them at least $k-1$ or K distinct elements according as the set contains 0 or 1 symbols in the $(1, 0, 1\ldots, 1)-$SDR and hence a $(1, 0, 1, \ldots, 1)$ SDR exists for the sets $R_i^{**\prime}$. We arrange them in the second column such that the 2nd position is blank and the rest of the positions are filled by 1, 3,..., v each of the treatments occurring exactly once. We proceed in a similar manner till all the v columns are filled and the design thus obtained is our required design.

The designs for $v=6$, 8, and 10 are given below :

$v=6$

x	3	4	5	6	2
3	x	5	6	1	4
2	6	x	1	4	5
5	1	6	x	2	3
6	4	2	3	x	1
4	5	1	2	3	x

$v=8$

x	3	4	5	6	7	8	2
3	x	5	6	7	8	1	4
4	6	x	7	8	1	2	5
5	8	1	x	2	3	6	7
6	4	7	8	x	2	3	1
7	1	8	2	4	x	5	3
8	5	2	3	1	4	x	6
2	7	6	1	3	5	4	x

$v=10$

x	3	4	5	6	7	8	9	10	2
3	x	5	6	7	8	9	10	1	4
4	5	x	7	8	9	10	1	2	6
5	6	7	x	9	10	1	2	3	8
6	7	8	10	x	1	2	3	4	9
2	1	9	8	10	x	3	4	7	5
8	9	10	1	2	4	x	6	5	3
7	10	2	9	3	5	4	x	6	1
10	8	1	3	4	2	6	5	x	7
9	4	6	2	1	3	5	7	8	x

We can easily compare the efficiency of this class of designs with the corresponding balanced incomplete block design with rows as blocks. After some routine calculations the estimate of this efficiency comes to

$$\text{Efficiency}=\frac{(v-1)\text{ column } MS+(v^2-3v+1)\text{ Error}}{v(v-2)\text{ Error } MS}$$

B. *Another class of balanced designs*

Let a, b and t be positive integers such that $a+(u-1)\,b=vt$. Then the design with incidence matrices $N=(a-b)I_u+bJ_u$, $L=t\,J_{v\,u}=M$, where $J_{v,u}$ is a vxu matrix with $+1$ everywhere can also be easily seen to be a balanced design for the row, column and treatment comparisons and this series belong to class 1 designs of the three factor additive designs of Pothoff (1962). For given values of a, b, u and v these designs can be trivially constructed. For $u=3=u'$, $v=4$, $a=0$, $b=2$, we have the design

x	1, 2	3, 4
3, 4	x	1, 2
1, 2	3, 4	x

For $u=3=u'$, $v=5$, $a=1$, $b=2$, we have the design

1	2, 3	4,5
3, 5	4	1, 2
2, 4	1, 5	3

For $u=3=u'$ $v=5$, $a=2$, $b=4$, we have the design

1, 2	3, 4, 5, 1	2, 3, 4, 5
3, 4, 5, 1	2, 3	4, 5, 1, 2
2, 3, 4, 5	2, 4, 5, 1	1, 3

REFERENCES

1. Aggarwal, Hira Lal (1966) : Two way elimination of heterogeneity. Calcutta Stat. Assn. Bull, 15, 32-38.
2. Aggarwal, Hira Lal (1966 *b*) : Some systematic methods of constructions of designs for two way elimination of heterogeneity. Calcutta Stat. Assn. Bull, 15, 93-108
3. Aggarwal, Hira Lal (1966 *c*) : Some methods of construction of designs for two-way elimination of heterogeneity—I, Jour. Amer. Stat. Assn., 61, 1153-1171.
4. Aggarwal, Hira Lal (1966 *d*) : Some generalizations of distinct representatives with applications to statistical designs. Ann. Math. Stat. 37, 525-528.
5. Freeman, G. H. and Jeffers, J.N.R. (1962) : Estimation of means and standard errors in the analysis of experiments by electronic computers. Jour. Roy. Stat. Soc. Ser. B, 24, 435-446.
6. Pothoff, R.F. (1962 *a*) : Three factor additive designs more general than the latin square. Technometrices, 4, 187-208.